Mesonic Width Effects on the Momentum Dependence of the $\rho - \omega$ Mixing Matrix Element

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Abstract

It is shown, in a model independent way, that the large difference in ρ and ω widths gives rise to a new source of momentum dependence for the $\rho - \omega$ mixing matrix element. The q^2 dependence arising due to the meson widths leads to a significant alteration of the result obtained in the zero-width approximation usually discussed in the literature. The origin of this strong momentum dependence lies in the difference between the ρ and ω meson widths.

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Experimentally, the mixing of ρ and ω mesons has been observed in the G-parity forbidden decays of the ω meson, $\omega \to \pi^+\pi^-$. The generally accepted value for the mixing matrix element is $\langle\rho|H_{\mathrm{csb}}|\omega\rangle = -0.00452 \pm 0.00006$ GeV^2 [1]. This value is extracted at $q^2 \simeq m_\omega^2$ and is often used to generate the NN potential in a meson exchange approach [2,1]. Even though the ρ and ω mesons are off-shell, it is common to find the use of the on-shell value. Thus there is an implicit assumption that the mixing matrix element is q^2 independent.

Recently, it has been argued that the $\rho - \omega$ mixing matrix element, extracted from the on-mass-shell mesons, should change significantly off-shell [3–10]. This off-shell behavior significantly alters the contribution of $\rho - \omega$ mixing to the CSB NN potential and its effects in $n-p$ scattering [11]. Theoretical calculations of the off-shell variations of the $\rho-\omega$ mixing matrix element have used various models that include mixing through $q\bar{q}$ loops [3,4], $N\overline{N}$ loops [5], a QCD sum rule calculation [6,7], and a hybrid quark-meson coupling model [8,9]. In all of these calculations, the ρ and ω mesons are treated as stable particles and their decay widths are neglected.

In this Letter, we shall show, in a model independent way, that the large difference in ρ and ω widths ($\Gamma_\rho = 151.5$ MeV, $\Gamma_\omega = 8.4$ MeV) gives rise to a new source of momentum dependence for the $\rho - \omega$ mixing matrix element. The q^2 dependence arising due to the meson widths leads to a significant alteration of the result obtained in the zero-width approximation, typically discussed in the literature [3–8]. Thus in our view, any discussion of the q^2 dependence of the $\rho - \omega$ mixing matrix element that does not include the finite width effects of the ρ and ω mesons is incomplete.

Let us start from the ρ and ω mixed propagator [6]

$$\Pi_{\mu\nu}^{\rho\omega}(q) = i \int d^4x e^{iq \cdot x}\langle 0|T\rho_\mu(x)\omega_\nu(0)|0\rangle \equiv -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi^{\rho\omega}(q^2) \,, \tag{1}$$

where ρ_μ and ω_ν are interpolating fields representing the ρ and ω mesons, respectively. The analytic structure of the mixed propagator allows us to write a dispersion relation of the form,

$$\mathrm{Re}\,\Pi^{\rho\omega}(q^2) = \frac{P}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\,\Pi^{\rho\omega}(s)}{(s - q^2)}ds \,, \tag{2}$$

which is valid to leading order in quark masses [6].

The mixing matrix element $\theta(q^2)$ has the following definition [6]

$$\mathrm{Re}\,\Pi^{\rho\omega}(q^2) \equiv \frac{\theta(q^2)}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)} \,. \tag{3}$$

Traditionally $\theta(q^2)$ is regarded as mixing between ρ and ω ground states. However, Eq. (2) indicates that $\theta(q^2)$ must also include physics of excited states. Hence to make contact with the traditional phenomenology, one must select ρ and ω interpolating fields which have maximal overlap with the ground state mesons.

In fact, it has been argued recently that there is no unique choice of the interpolating fields ρ_μ and ω_ν and the mixed propagator depends on the choice of interpolating fields [12,13]. Although this is certainly the case, there is, however, an obvious and physically

reasonable choice which is consistent with standard traditional phenomenology. Namely, one should select interpolating fields with maximal overlap with ground states. One could introduce wave functions to smear out the relative separation of the quark field operators in the interpolating fields to improve the overlap with ground states as done in lattice calculations. However, such a nonlocal approach is not gauge invariant. Alternatively one might consider interpolating fields involving gluon field strength operators or derivatives. However, such interpolating fields are of higher dimension and have an increased overlap with excited states relative to the ground states. Hence the preferred interpolating fields most consistent with the traditional phenomenology are local and of minimal dimension. The standard vector currents satisfy these criteria.

To illustrate the effect of including the finite widths of the mesons on the mixing matrix element, it is sufficient to saturate the imaginary part of the mixed propagator on the right hand side of (2) with ρ and ω mesons alone. We begin with the zero-width approximation,

$$\text{Im}\,\Pi^{\rho\omega}(s) = \pi\,F_\rho\,\delta(s - m_\rho^2) - \pi\,F_\omega\,\delta(s - m_\omega^2)\,. \tag{4}$$

Here F_ρ and F_ω denote the coupling strengths of the interpolating fields to the physical meson states. While a more sophisticated treatment of the spectral density or dispersion relation might be desirable, such complications unnecessarily obscure the qualitative physics we are emphasizing here. Equation (2) now takes the form

$$\text{Re}\,\Pi^{\rho\omega}(q^2) = \frac{-F_\rho(q^2 - m_\omega^2) + F_\omega(q^2 - m_\rho^2)}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)}\,, \tag{5}$$

which implies

$$\theta(q^2) = \frac{1}{2}\,(F_\rho + F_\omega)\,\delta m^2 - (F_\rho - F_\omega)(q^2 - m^2)\,. \tag{6}$$

Here we have introduced the notation $\delta m^2 \equiv m_\omega^2 - m_\rho^2$ and $m^2 \equiv (m_\omega^2 + m_\rho^2)/2$. One may then express $\theta(q^2)$ in terms of $\theta(m^2) = (F_\rho + F_\omega)\,\delta m^2/2$ as

$$\theta(q^2) = \theta(m^2)\left[1 + \lambda\,(\frac{q^2}{m^2} - 1)\right]\,, \tag{7}$$

where $\lambda \equiv -m^2(F_\rho - F_\omega)/\theta(m^2)$, and $\theta(m^2)$ is the on-shell $\rho - \omega$ mixing matrix element. Note that the q^2 dependence of the mixing matrix element arises due to the second term of Eq. (6). If $F_\rho = F_\omega$, implying $\lambda = 0$, then there is no q^2 dependence. Also note that the sign of $\theta(q^2)$ changes at $q^2 = m^2(\lambda - 1)/\lambda$. Hatsuda $et\ al.$ [6] have extracted the parameter λ from a QCD sum rule analysis and quoted the values $1.43 \leq \lambda \leq 1.85$. While we have a few reservations regarding their analysis, we will use their estimate of λ to illustrate the effects of mesonic widths and their relative importance in determining the q^2 dependence of $\rho - \omega$ mixing. We will also consider other values to more clearly illustrate our findings. In Fig. 1 we have plotted the ratio $\theta(q^2)/\theta(m^2)$ for various values of λ.

In the above discussion, we have assumed zero-width sharp poles for the mesons. In nature, however, both ρ and ω are resonances with finite widths. To include these finite widths, we replace the δ-functions in Eq. (4) by a normalized Breit-Wigner form
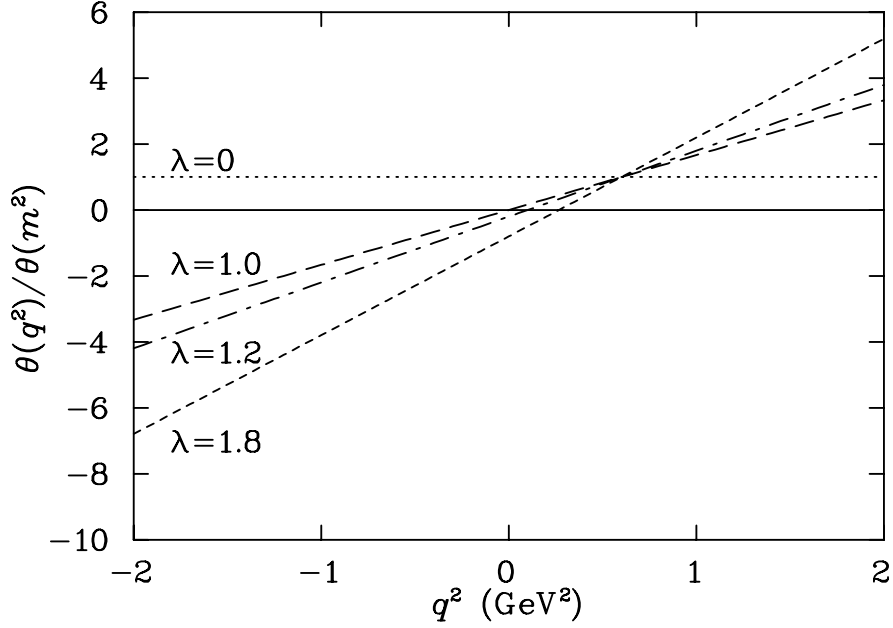
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FIG. 1. The normalized $\rho - \omega$ mixing amplitude $\theta(q^2)/\theta(m^2)$ plotted as a function of q^2 for various values of λ.

$$\frac{1}{\pi} \frac{m\,\Gamma}{(q^2 - \overline{m}^2)^2 + m^2\,\Gamma^2} \, , \tag{8}$$

where $\overline{m}^2 = (m^2 - \Gamma^2/4)$, and Γ is the half width of the meson. This form has a smooth limit to the δ-function. In the absence of a solution to QCD, the precise shape of the spectral density is unknown. As such, we consider a sum of the ρ and ω resonant peaks with finite widths, which should provide a physically reasonable model for the spectral density. With the following integral definition

$$I_{\rho,\omega}(q^2) \equiv \frac{m_{\rho,\omega}\Gamma_{\rho,\omega}}{\pi} P \int_{4m_\pi^2}^{\infty} \frac{ds}{(s - q^2)\left[(s - \overline{m}_{\rho,\omega}^2)^2 + m_{\rho,\omega}^2\,\Gamma_{\rho,\omega}^2)\right]} \, , \tag{9}$$

we can write the analogue of Eq. (5) for Re $\Pi^{\rho\omega}$ in a compact form [14]

$$\text{Re } \Pi^{\rho\omega}(q^2) = F_\rho\,I_\rho(q^2) - F_\omega\,I_\omega(q^2) \, . \tag{10}$$

This expression reduces to the zero-width result of (5) in the limit of the meson widths going to zero.

Let us first adopt the simple zero-width form of Eq. (3) to define the mixing matrix element $\theta_1(q^2)$

$$\text{Re } \Pi^{\rho\omega}(q^2) = \frac{\theta_1(q^2)}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)} \, , \tag{11}$$

where Re $\Pi^{\rho\omega}(q^2)$ is now given by Eq. (10). This allows an examination of how the inclusion of finite meson widths affects the traditionally defined $\theta(q^2)$. With the definition $G_0(q^2) \equiv (q^2 - m_\rho^2)(q^2 - m_\omega^2)$ we have

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$$\theta_1(q^2) = G_0(q^2) \left[F_\rho\, I_\rho(q^2) - F_\omega\, I_\omega(q^2) \right] . \tag{12}$$

One can rewrite $\theta_1(q^2)$ in terms of $\theta_1(m^2)$ as

$$\theta_1(q^2) = \theta_1(m^2) \left\{ \frac{G_0(q^2)}{G_0(m^2)} \frac{I_\rho(q^2) - I_\omega(q^2)}{I_\rho(m^2) - I_\omega(m^2)} + \lambda\, \frac{G_0(q^2)}{m^2} \frac{I_\rho(q^2)I_\omega(m^2) - I_\omega(q^2)I_\rho(m^2)}{I_\rho(m^2) - I_\omega(m^2)} \right\}, \tag{13}$$

where $\lambda \equiv -m^2\,(F_\rho - F_\omega)/\theta_1(m^2)$, with $\theta_1(m^2)$ the on-shell value. Now both terms in braces are q^2 dependent. This arises because the widths for ρ and ω mesons are very different, and this leads to a different q^2 dependence in the integrals I_ρ and I_ω. In Fig. 2 we have plotted $\theta_1(q^2)/\theta_1(m^2)$ as a function of q^2 for various values of λ, with the physical values for the meson widths. A comparison with Fig. 1 reveals significant alteration of the q^2 dependence for all values of λ.

To be fully consistent, meson widths should also be included in extracting the mixing matrix element from $\Pi^{\rho\omega}$ and we relate the mixing matrix element to $\Pi^{\rho\omega}(q^2)$ by

$$\Pi^{\rho\omega}(q^2) \equiv \frac{\theta_\Gamma(q^2)}{\left[q^2 - (m_\rho - i\frac{\Gamma_\rho}{2})^2 \right] \left[q^2 - (m_\omega - i\frac{\Gamma_\omega}{2})^2 \right]} , \tag{14}$$

where Γ_ρ and Γ_ω are q^2-dependent. In particular, below threshold ($q^2 = 4m_\pi^2$), $\Gamma_{\rho,\omega} = 0$ and $\Pi^{\rho\omega}(q^2 < 4m_\pi^2)$ is real as it should be. Above threshold, we take Γ_ρ and Γ_ω to be constant for simplicity. Here we have introduced the notation θ_Γ to indicate the fully consistent definition of the mixing matrix element. The real part $\Pi^{\rho\omega}$ is now given by
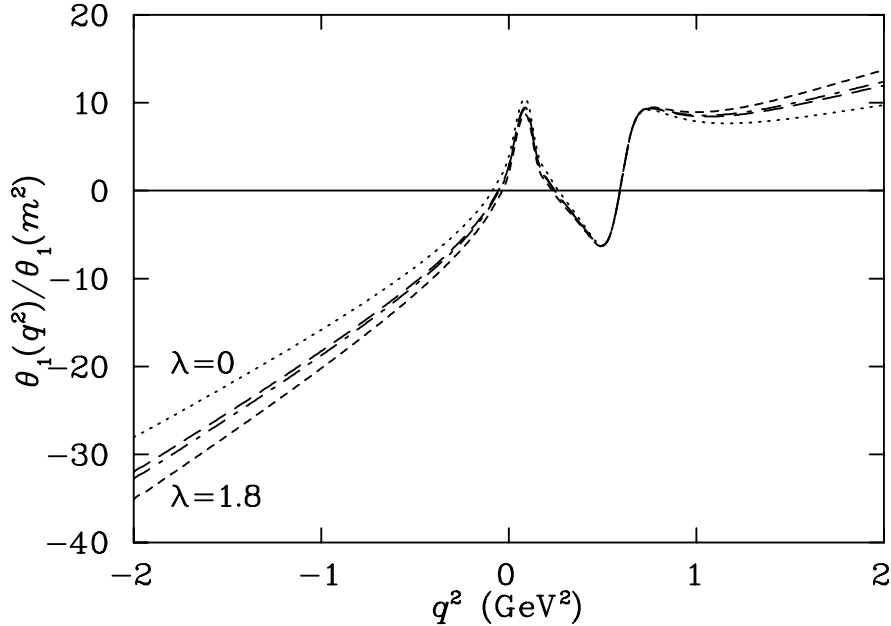

FIG. 2. $\theta_1(q^2)/\theta_1(m^2)$ as a function of q^2 for the same values of λ as used in Fig. 1, with $\Gamma_\rho = 151.5$ MeV and $\Gamma_\omega = 8.4$ MeV.
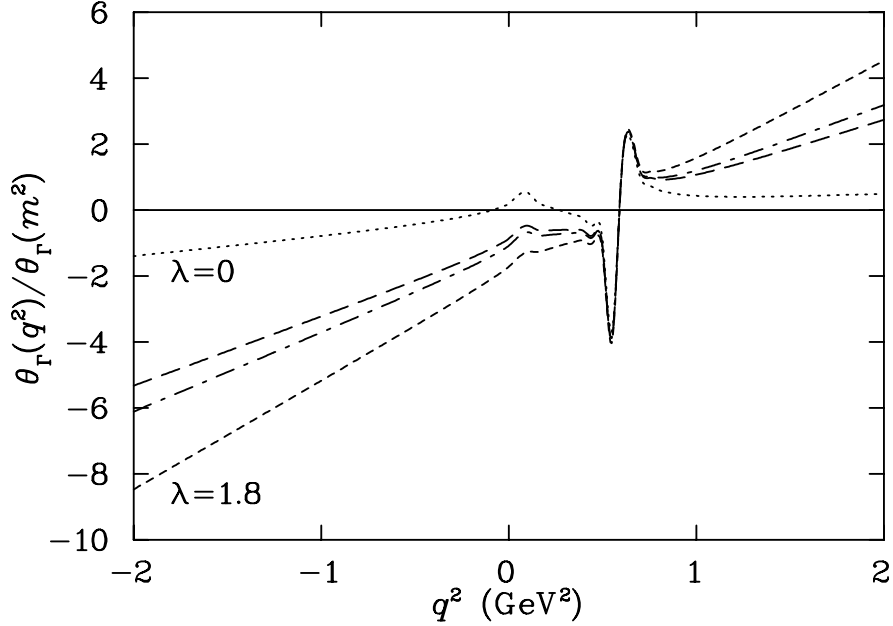
FIG. 3. $\theta_\Gamma(q^2)/\theta_\Gamma(m^2)$ as a function of q^2 for values of $\lambda = 0$, 1.0, 1.2, and 1.8, with $\Gamma_\rho = 151.5$ MeV and $\Gamma_\omega = 8.4$ MeV.

$$\text{Re } \Pi^{\rho\omega}(q^2) = \theta_\Gamma(q^2) \frac{(q^2 - \overline{m}_\rho^2)(q^2 - \overline{m}_\omega^2) - m_\rho m_\omega \Gamma_\rho \Gamma_\omega}{\left[(q^2 - \overline{m}_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2\right]\left[(q^2 - \overline{m}_\omega^2)^2 + m_\omega^2 \Gamma_\omega^2\right]} \ . \tag{15}$$

Recall, $\overline{m}_{\rho,\omega}^2 = (m_{\rho,\omega}^2 - \Gamma_{\rho,\omega}^2/4)$. As before, we can write $\theta_\Gamma(q^2)$ in terms of $\theta_\Gamma(m^2)$ as

$$\theta_\Gamma(q^2) = \theta_\Gamma(m^2)\left\{\frac{G(q^2)}{G(m^2)} \frac{I_\rho(q^2) - I_\omega(q^2)}{I_\rho(m^2) - I_\omega(m^2)} + \lambda \frac{G(q^2)}{m^2} \frac{I_\rho(q^2)I_\omega(m^2) - I_\omega(q^2)I_\rho(m^2)}{I_\rho(q^2) - I_\omega(q^2)}\right\}, \tag{16}$$

where $G(q^2)$ is defined as

$$G(q^2) = \frac{\left[(q^2 - \overline{m}_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2\right]\left[(q^2 - \overline{m}_\omega^2)^2 + m_\omega^2 \Gamma_\omega^2\right]}{(q^2 - \overline{m}_\rho^2)(q^2 - \overline{m}_\omega^2) - m_\rho m_\omega \Gamma_\rho \Gamma_\omega} , \tag{17}$$

and $\lambda \equiv -m^2(F_\rho - F_\omega)/\theta_\Gamma(m^2)$, with $\theta_\Gamma(m^2)$ the on-shell value. We note that the relation of (14) provides better contact with the experimentally extracted on-shell value than that given in (3). A plot of $\theta_\Gamma(q^2)$ for various values of λ is shown in Fig. 3. The q^2 dependence of $\theta_\Gamma(q^2)$ is softer than that for $\theta_1(q^2)$. We observe that the expression for $\theta_\Gamma(q^2)$ in (16) is q^2 dependent regardless of the value taken for λ. This is in accord with the arguments of Ref. [13] based on unitarity and analyticity.

Recently, arguments have been made for the vanishing of $\Pi^{\rho\omega}(q^2)$ at q^2. The arguments are quite general and apply to any effective field theory in which there is no explicit mass-mixing term in the bare Lagrangian. Unfortunately, we are unable to comment on the viability of such models without knowledge of the value for λ. A fundamentally based QCD determination of λ is forthcoming [16].
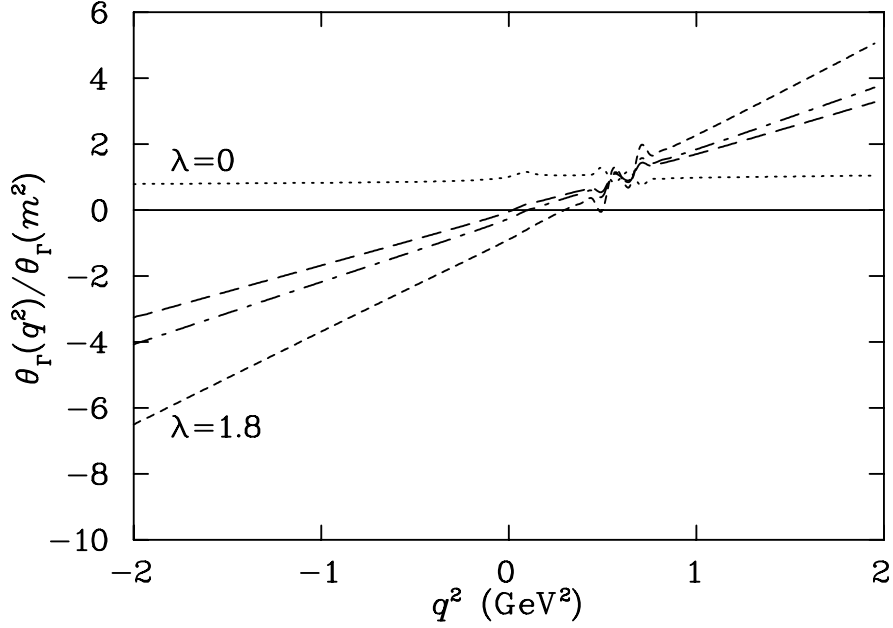
FIG. 4. $\theta_\Gamma(q^2)/\theta_\Gamma(m^2)$ as a function of q^2, with $\Gamma_\rho = \Gamma_\omega = 100$ MeV. The values of λ are the same as those used in previous figures.

For comparison, we present in Fig. 4 the ratio $\theta_\Gamma(q^2)/\theta_\Gamma(m^2)$ for the same values of λ as in Fig. 3, but with $\Gamma_\rho = \Gamma_\omega = 100$ MeV. The q^2 dependence in this case is similar to that for the zero-width case.

The effects of including an energy dependent ρ-meson width [15] on the right-hand side of the dispersion relation of (2) are illustrated in Fig. 5. Here we adopt the standard form [15]

$$\Gamma_\rho(s) = \Gamma_\rho(m_\rho^2) \frac{m_\rho}{\sqrt{s}} \left(\frac{s - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2}, \quad s \geq 4m_\pi^2, \tag{18}$$

normalized to the previous value at $s = m_\rho^2$. The predominant effect is a smoothing of the curves in the threshold regime. The oscillation seen in Fig. 3 is an artifact of the abrupt onset of the ρ-meson width at the threshold $4\,m_\pi^2$ as discussed following Eq. (14). The energy dependence of the ρ width does not alter our general conclusions.

In summary, we have shown in a model independent way that the inclusion of ρ and ω widths significantly alters the q^2 dependence of the $\rho - \omega$ mixing matrix element and hence of the mixed meson propagator. This behavior arises from the fact that the widths of ρ and ω are different. Any model calculation addressing the q^2 dependence of the $\rho - \omega$ mixing matrix element that does not include meson width effects is incomplete.
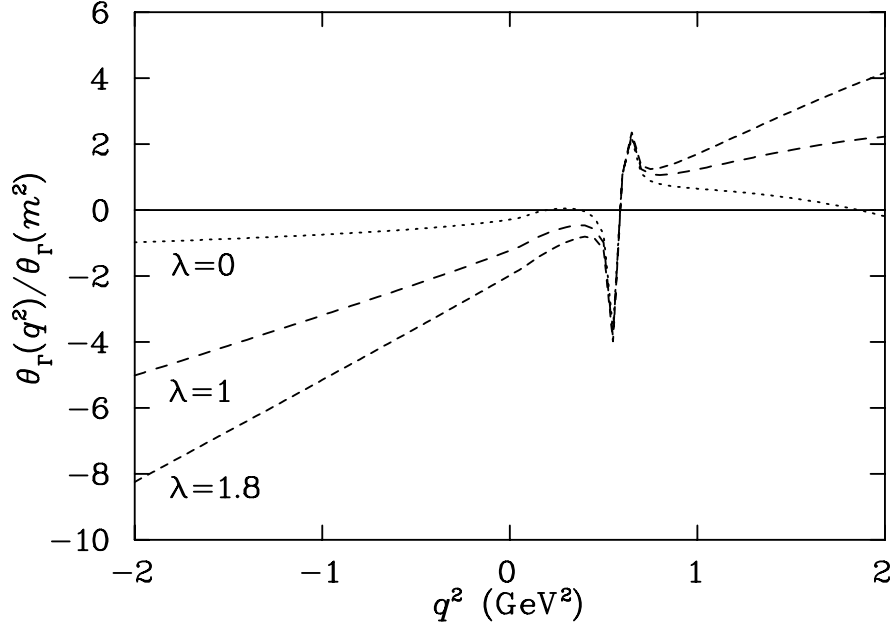
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FIG. 5. $\theta_\Gamma(q^2)/\theta_\Gamma(m^2)$ as a function of q^2 with an energy dependent width for the ρ meson as given in (18).

REFERENCES

[1] S.A. Coon and R.C. Barrett, Phys. Rev. C **36**, 2189 (1987).

[2] E.M. Henley and G.A. Miller, in *Mesons and Nuclei*, eds. M. Rho and D. Wilkinson, (North Holland, Amsterdam, 1979).

[3] T. Goldman, J. A. Henderson and A. W. Thomas, Few-Body Systems **12**, 123 (1992); Mod. Phys. Lett. **A7**, 3037 (1992).

[4] G. Krein, A. W. Thomas, and A. G. Williams, Phys. Lett. **B317**, 293 (1993).

[5] J. Piekarewicz and A. G. Williams, Phys. Rev. C **47**, R2461 (1993).

[6] T. Hatsuda, E. M. Henley, T. Meissner and G. Krein, Phys. Rev. C **49**, 452 (1994).

[7] K. Maltman, "The Mixed-Isospin Vector Current Correlator in Chiral Perturbation Theory and QCD Sum Rules", preprint ADP-95-20/T179, hep-ph/9504237, April 1995.

[8] K. L. Mitchell, P. C. Tandy, C. D. Roberts and R. T. Cahill, Phys. Lett. **B335**, 282 (1994).

[9] A. N. Mitra and K. C.Yang, preprint NUCL-TH-9406011, (1994).

[10] H. B. O'Connell, B. C. Pearce, A. W. Thomas and A. G. Williams, Phys. Lett. **B336**, 1 (1994); preprint ADP-95-15/T168, hep-ph/9503332, (1995).

[11] M.J. Iqbal and J.A. Niskanen, Phys. Lett. **B322**, 7 (1994).

[12] T. D. Cohen and G. A. Miller, "Rho-Omega Mixing and Charge Symmetry Breaking in the N-N Potential", preprint nucl-th/9506023.

[13] K. Maltman, "Two Model-Independent Results for the Momentum Dependence of $\rho - \omega$ Mixing", preprint ADP-95-35/T189, nucl-th/9506024.

[14] The integrals I_ρ and I_ω are cut off at $s = 3\,\mathrm{GeV}^2$. This value is selected as the integral values are stable and the physics of excited resonances is expected to alter the spectral density.

[15] H. Pilkuhn, *Relativistic Particle Physics* (New York, Springer-Verlag, 1979), Chapter 4.

[16] M.J. Iqbal, X. Jin, and D. B. Leinweber, in preparation.